then admittance across the plates neglecting fringing is

$$Y = j\omega\frac{A\epsilon_p}{d}$$

$$= \frac{A\epsilon_0}{d}\left\{\frac{\nu\omega_p^2}{\omega^2+\nu^2} + j\omega\left(1 - \frac{\omega_p^2}{\omega^2+\nu^2}\right)\right\}.$$

The equivalent circuit of this is clearly as shown in Fig. 1. C is the free space capacitance between the plates. When $\nu \ll \omega$, this circuit will give parallel resonance at $\omega = \omega_p$ and its $Q = \omega/\nu$.

Series resonance could be obtained by placing either a capacitance or an inductance in series with this circuit. In practice, the plates will have to be insulated from the plasma, thus adding a series capacitance C_1; the resultant circuit will show series resonance at $\omega < \omega_p$, resonant angular frequency being

$$\omega_r = \omega_p\sqrt{\frac{C}{C+C_1}}.$$

Since in practice C_1 is likely to be much greater than C, $\omega_r \to \omega_p\sqrt{C/C_1}$. This is the effect obtained by Yeung and Sayers.[2]

The effect of C_1 could be reduced by a small series capacitance and one could even make $\omega_r > \omega_p$ by adding a suitable series tuned circuit; however, this would seriously reduce the control of resonance by electron number density.

The bandwidth of such a circuit would depend on the relative value of generator and load resistance R_G and R_L, respectively. The ratio of center frequency to bandwidth is

$$Q = \frac{1}{\frac{\nu}{\omega_r} + \frac{(R_G+R_L)C\omega_p^2}{\omega_r}}$$

$$= \frac{1}{\frac{\nu}{\omega_r} + (R_G+R_L)C_1\omega_r}.$$

Clearly, in order to make this ratio large, C_1 and hence C must be small, *i.e.*, A should be small and d large.

Consider now the case of a plasma cylinder placed centrally across a waveguide and parallel to the broad side, the guide carrying a wave in TE_{10} mode. Bryant and Franklin[3] have shown that, neglecting the effect of glass wall, the admittance of such a plasma column placed in a matched guide is to a first approximation

$$Y_r = j2Y_0\frac{\pi r^2 k_0^2}{bk_g}\cdot\frac{\epsilon_p - \epsilon_0}{\epsilon_p + \epsilon_0}.$$

where r is the plasma radius b is narrow guide dimension and Y_0 is the characteristic admittance of the guide.

Applying the above given expression for ϵ_p, we get

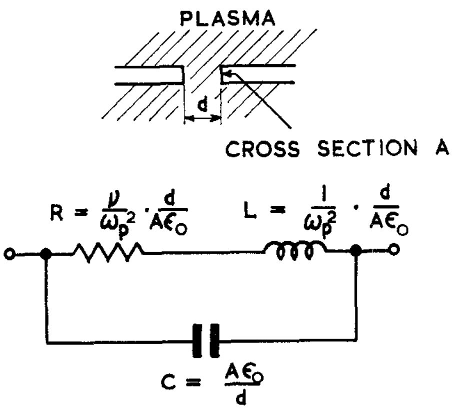

Fig. 1—The equivalent impedance of two electrodes immersed in plasma but insulated from it.

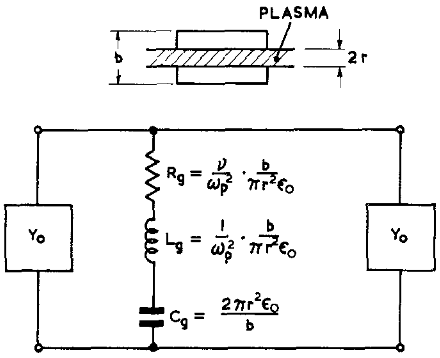

Fig. 2—The equivalent impedance of a plasma-filled tube across a waveguide excited in TE_{01} mode.

$$Y_r = \frac{\pi r^2\epsilon_0}{b}\cdot\frac{1}{\frac{\nu}{\omega_p^2} + j\frac{\omega}{\omega_p^2} + \frac{1}{2j\omega}}.$$

Hence the equivalent circuit is as shown in Fig. 2 where

$$R_g = \frac{\nu}{\omega_p^2}\cdot\frac{b}{\pi r^2\epsilon_0}, \qquad L_g = \frac{1}{\omega_p^2}\cdot\frac{b}{\pi r^2\epsilon_0},$$

$$C_g = \frac{2\pi r^2\epsilon_0}{b}.$$

This resonates at $\omega = \omega_p/\sqrt{2}$ and the Q of the series tuned circuit alone is ω/ν. The circuit produces maximum reflection and minimum transmission at resonance. Defining the bandwidth of reflected power as frequency interval within which reflected power is greater than $\frac{1}{2}$ maximum, we obtain the ratio of center frequency to bandwidth

$$Q_R = \frac{1}{\frac{\nu}{\omega} + \frac{\pi r^2 k_0^2}{bk_g}} = \frac{1}{\frac{\nu}{\omega} + \frac{2\pi^2 r^2\lambda_g}{b\lambda_0^2}}.$$

The second term in the denominator, the damping term, will usually be larger than ν/ω, typically in a standard S-band guide at 3 Gc/s it will be of the order of 0.1, giving $Q_R \approx 10$. The effect of glass walls is small. (See Bryant and Franklin.[3])

While it may be possible to obtain larger Q_R by suitable dimensional adjustments, it should be observed that at resonance, the reflected power will be proportional to $[1 - Q_R(\nu/\omega)]^2$ and the transmitted power to $[Q_R(\nu/\omega)]^2$. It may be difficult to make ω/ν greater than 1000; hence, if Q_R were made, *e.g.*, 100, only 80 per cent or so of power would be reflected at resonance, and nearly 20 per cent lost in heating the plasma. This would get worse if Q_R were increased still further.

Looking at the transmitted signal, we can define the bandwidth either as the frequency interval within which the attenuation is more than half maximum leading to Q_A, or the interval within which the transmitted power is less than twice minimum leading to Q_T.

It can be shown that $Q_A = Q_R$ while

$$Q_T = \frac{\omega}{\nu}\left\{1 - 2\left(\frac{bk_g}{\pi r^2 k_0^2}\cdot\frac{\nu\omega}{\omega_p^2}\right)^2\right\},$$

which for very small collision frequency ($\nu \ll \omega$) tends to ω/ν.

Finally we would like to point out that the term Q is used somewhat loosely both here and by other authors. The loaded Q of the circuit in Fig. 2 calculated on the basis of stored and dissipated energy comes to

$$\frac{1}{\frac{\nu}{\omega}\left(1 + 2\frac{bk_g}{\pi r^2 k_0^2}\cdot\frac{\nu\omega}{\omega_p^2}\right)}$$

which is like neither Q_R, Q_A nor Q_T.

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[2] T. H. Yeung and J. Sayers, *Proc. Phys Soc. (London)*, vol. B70, pp. 663–668; 1957.

[3] G. H. Bryant and R. N. Franklin, *Proc. Phys. Soc. (London)*, vol. 81, pp. 531–543 and 790–792; 1963.

Intrinsic Attenuation

In connection with a recent paper by Beatty,[1] we wish to advise that we have found a general method of decomposition of the two-arm dissipative waveguide junctions taking into account the intrinsic attenuation.[2]

We have defined for any two-arm junction D_1 a family of associated two-arm junctions so that each of its element D_2 verifies

$$q_1 d_1 q_2 = d_2$$

q_1, q_2, d_1, d_2 being the wave transformation matrices of the two-arm junctions Q_1Q_2 D_1D_2 (Q_1, Q_2 being nondissipative). Moreover, to study a family, we have found it easier to consider the matrices

$$d_r = \begin{vmatrix} c_{11}^* & -c_{21}^* \\ -c_{12}^* & c_{22}^* \end{vmatrix}$$

[1] R. W. Beatty, "Intrinsic attenuation," IEEE Trans. on Microwave Theory and Techniques, vol. MTT-11, pp. 179–182; May, 1963.

[2] "Classification des quadripoles dissipatifs en hyperfréquence," *Compt. rend. acad. sci.*, vol. 257, p. 3576; 1963.

(c_{ij} being the elements of the transfer matrix of a two-arm junction) and

$$P = dd_r.$$

We have pointed out that to have D_1 and D_2 belonging to the same family it is necessary and sufficient that the corresponding matrices P verify

$$P_2 = q_1 P_1 q_1^{-1}.$$

Therefore, all the elements of the family have the same equation:

$$\det. (\lambda I - P) = \lambda^2 - [|c_{11}|^2 + |c_{22}|^2 - |c_{12}|^2 - |c_{21}|^2]\lambda + 1 = 0,$$

and the expression

$$T = |c_{11}|^2 + |c_{22}|^2 - |c_{12}|^2 - |c_{21}|^2$$

is an invariant in the family.

1) $T>2$ is the case that Beatty[1] has considered. By the diagonalization of P, the intrinsic losses are easily set up.
2) $T=2$ cannot be diagonalized. One can decompose the two-arm junction into a complex admittance preceded and followed by nondissipative two-arm junctions (one of them being a length of lossless waveguide), and if this two-arm junction is symmetric, it can be decomposed into a complex admittance preceded and followed by two equal lengths of lossless waveguide. In this case, we can theoretically reduce the insertion loss to a value as small as we want.

These notions will be generalized to the m-arm junctions in a later paper.

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Correction to "Superheterodyne Radiometers for Use at 70 Gc and 140 Gc"[1]

In Appendix II of the above paper, (9) and (10) should read

$$\omega_s = 2\omega_c + \omega_{IF} \quad (9)$$

$$\omega_i = 2\omega_c - \omega_{IF}. \quad (10)$$

Each n in (15) and (17) should be an η; and, in the equation immediately above (18), the large curly bracket should be an integral sign.

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[1] R. Meredith and F. L. Warner, "Superheterodyne radiometers for use at 70 Gc and 140 Gc," IEEE Trans. on Microwave Theory and Techniques, vol. MTT-11, pp. 397–411; September, 1963.

Measurements of Varactor Diode Impedance

Summary

Impedance measurements of varactor diodes have been made on a slotted line in the frequency range 2–18 Gc. The experimental technique is described, and a method for deriving a simple lumped equivalent circuit is shown. Typical circuit values are quoted.

Introduction

In earlier microwave measurements of varactor diodes, *e.g.*, by Houlding,[1] Harrison,[2] and Mavaddat,[3] the work has been done in such a way that the circuit values derived have been those of the actual semiconductor junction, rather than over-all values for the complete encapsulated diode. In general, the method used involved matching the varactor diode and its mount to the waveguide or line and studying the variations in impedance at a chosen reference plane as the bias was altered. The measurements have usually been made at a single frequency.

This approach to the problem has difficulties; there are, for example, uncertainties due to losses in matching components and in the correct choice of reference planes. It is nevertheless quite helpful to varactor diode manufacturers as the technique gives results which apply to the semiconductor junction alone, regardless of the encapsulation.

The microwave circuit designer has, however, almost the opposite problem. He is particularly concerned with the impedance properties of the encapsulated varactor package for various ranges of frequencies. The method described in this communication was developed with this purpose in mind and the results should be of particular interest to the designers of parametric amplifiers.

A direct microwave measurement technique was employed, using normal methods for measuring high mismatch on a slotted line. The results showed that the varactors could be successfully represented by a simple lumped constant circuit, with circuit values close to those quoted by the manufacturers for the junction.

Measurement Technique

The physical form of the diodes measured is as shown in Fig. 1(a). In early experiments, the diode was mounted in a simple collet, and measured on a 50-ohm slotted line, with an N-type connector between the diode and the line. The results of these experiments showed that the connector and the collet caused an appreciable perturbation of the results, and a simpler mounting arrangement was devised. This is shown in Fig. 1(b). The diode was mounted directly at the end of the 50-ohm slotted line, being held in position by gentle pressure from the inner conductor. In this arrangement the diameter of the inner conductor of the line is the same, or very nearly the same, as the diameter of the end caps of the diodes. This avoids discontinuity capacitances which might be confused with the stray capacity of the diode itself. A reference plane was chosen 0.020 inch inside the diode as shown in Fig. 1(b), and a short circuit could be provided across the line at this plane, which corresponds to the inner edge of the brass cap. When this short circuit was in place, the position of this reference plane, with respect to the scale of the slotted line, could be found by measuring positions of minima.

Having established the reference plane, impedance measurements of the diodes could be made by normal slotted line methods. However, several precautions had to be taken.

First, the input power had to be kept below a certain level, as the impedance of the varactor diode was affected by power level. This upper limit of power was that, giving a voltage of 1 mv across the diode. A convenient way of ensuring that the power was low enough was to use the slotted line method in which source and detector of the slotted line are interchanged.[1] In this method, the load and the detector operate at comparable power levels; in some other methods the power level at the probe is some 20 or 30 db below that at the load and it is more difficult to obtain sufficient sensitivity.

Second, the VSWR of the diodes was very high. Because of this, the double minimum method[4] of impedance measurement was used. In this method, measurement is made of the distance Δx between the two symmetrical carriage positions about a minimum for which the detector reading is twice the minimum reading. For a square-law detector, provided $S>10$

$$\text{VSWR} \qquad S = \frac{\lambda}{\pi \Delta x}$$

where λ is the wavelength. This method is better for large VSWR values than the conventional one, which would demand calibration of the detector system over a wide dynamic range.

When this method is used with source and detector in the normal positions, the errors due to probe coupling are very low, as the probe is operated in a region of low impedance. Similarly with the probe-feed arrangement, tight probe coupling will introduce little error, as the impedance seen by the probe in the region of the minimum differs little from the probe capacitance only. This gives the constant-current source that this method requires to be accurate. Fig. 2 shows the experimental arrangement. An unmodulated source was used, and the detector output was measured on a valve voltmeter (V.V.M).

Results

Fig. 3, page 470, shows some typical results obtained with diode "A." The line is drawn dashed beyond the 12-Gc point since

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[1] N. Houlding, "Measurement of varactor quality," *Microwave J.*, vol. 3, p. 40; 1960.

[2] R. I. Harrison, "Parametric diode Q measurements," *Microwave J.*, vol. 3, p. 43; 1960.

[3] R. Mavaddat, "Diode Q-Factor Measurements," University College of North Wales School of Applied Science; February, 1963.

[4] E. L. Ginzton, "Microwave Measurements," McGraw-Hill Book Company, Inc., New York, N. Y., p. 266; 1957.